Getting Pass Tutorial for DSE

Name: ____________________

Class: __________

Tutorial 11 Linear Inequalities

Key Points

Basic Properties of Inequalities

(a) Transitive property

If $a > b$ and $b > c$, then $a > c$.

(b) Additive property

If $a > b$, then

(i) $a + c > b + c$,

(ii) $a - c > b - c$.

(c) Multiplicative property

(i) If $a > b$ and $c > 0$, then $ac > bc$.

(ii) If $a > b$ and $c < 0$, then $ac < bc$.

(d) Reciprocal property

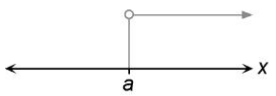
(i) If $a > b > 0$, then $\frac{1}{a} < \frac{1}{b}$.

(ii) If $a < b < 0$, then $\frac{1}{a} > \frac{1}{b}$.

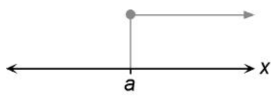
Note: These properties still hold when '>' and '<' are replaced by '≥' and '≤' respectively.

Graphical Representations of Solutions of Linear Inequalities

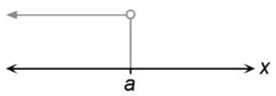
The solutions of a linear inequality can be represented graphically on a number line.



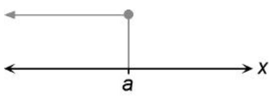
$x > a$



$x \geq a$



$x < a$



$x \leq a$

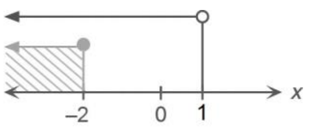
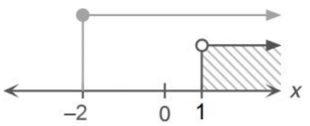
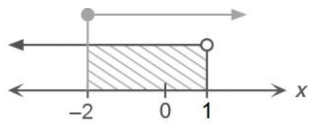
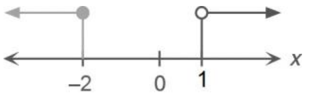
Note: 1. The hollow circle '○' means that 'a' is not included.

2. The solid circle '•' means that 'a' is included.

Compound Linear Inequalities

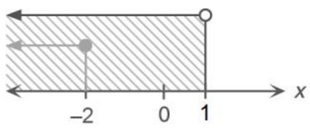
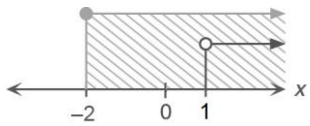
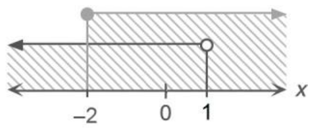
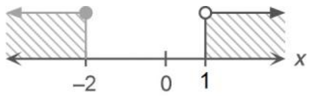
(a) Connected by 'and'

The solutions for this kind of inequality must satisfy all the inequalities.

Compound inequality	Solutions
$x \le -2$ and $x < 1$ 	$x \le -2$
$x \ge -2$ and $x > 1$ 	$x > 1$
$x \ge -2$ and $x < 1$ 	$-2 \le x < 1$
$x \le -2$ and $x > 1$ 	no solutions

(b) Connected by 'or'

The solutions for this kind of inequality must satisfy at least one of the inequalities.

Compound inequality	Solutions
$x \le -2$ or $x < 1$ 	$x < 1$
$x \ge -2$ or $x > 1$ 	$x \ge -2$
$x \ge -2$ or $x < 1$ 	all real values of x
$x \le -2$ or $x > 1$ 	$x \le -2$ or $x > 1$

Revision Exercise

Short Questions

1. **(a)** Solve the inequality $x - 3 > \dfrac{2x + 24}{7}$.

(b) Represent the solutions of (a) on a number line.

2. **(a)** Solve the inequality $\dfrac{18x - 25}{8} \ge 2x$.

(b) Write down the smallest integer that satisfies the inequality in (a).

3. **(a)** Solve the inequality $\frac{23x}{4} \le 5x - 8$.

(b) Write down the greatest integer that satisfies the inequality in (a).

4. **(a)** Solve the inequality $-5x - 7 \le 2 - 3x$.

(b) Find the range of values of x which satisfy both the inequalities $-5x - 7 \le 2 - 3x$ and $3x - 8 < 0$.

5. **(a)** Solve the inequality $6 + 5x < 4(x + 2)$.

(b) Solve the compound inequality '$6 + 5x < 4(x + 2)$ or $2x \le 10$'.

6. **(a)** Solve the inequality $\frac{22 + 3x}{7} \le 8 + 3x$.

(b) Find the number of integers satisfying both the inequalities $\frac{22 + 3x}{7} \le 8 + 3x$ and $15 - 3x > 0$.

7. **(a)** Solve the inequality $\dfrac{-5x-4}{3} \geq 2x+3$.

(b) Write down all integers which satisfy both the inequalities $\dfrac{-5x-4}{3} \geq 2x+3$ and $4x+20 \geq 0$.

8. **(a)** Solve the inequality $\dfrac{4-3x}{2} < 3x-7$.

(b) Write down the smallest integer which satisfies the solutions of '$\dfrac{4-3x}{2} < 3x-7$ or $4x+16 \geq 0$'.

9. **(a)** Solve the compound inequality '$\dfrac{6+5x}{4} \geq 3(x-3)$ or $7x-24 < 0$'.

(b) Write down the greatest integer which satisfies the solutions found in (a).

10. **(a)** Find the range of values of x which satisfy both the inequalities $3(x+4)<\frac{5-4x}{8}$ and $3x+18\geq 0$.

(b) How many negative integers satisfy both the inequalities in (a)?

11. Consider the compound inequality

$2x-5>7(2x-9)$ or $2x+8\geq 0$ …… (*)

(a) Solve (*).

(b) Find the smallest positive integer satisfying (*).

12. Consider the compound inequality

$x-7\leq 3(2x+6)$ or $x+7<0$ …… (*)

(a) Solve (*).

(b) Find the number of integers not satisfying the solutions of (*).

Multiple Choice Questions

1. The solutions of $7(2x-3)<21$ are

A. $x<3$.
B. $x>3$.
C. $x<0$.
D. $x>0$.

2. The solutions of $3(2-3x)-10\le 14$ are

A. $x\le -2$.
B. $x\ge -2$.
C. $x\le -6$.
D. $x\ge -6$.

3. The solutions of $22\le 3(x+4)+2x$ are

A. $x\le -2$.
B. $x\ge -2$.
C. $x\le 2$.
D. $x\ge 2$.

4. The solutions of $4x\le -18-5x$ and $20-5x\ge 0$ are

A. $x\le -2$.
B. $x\le 4$.
C. $-2\le x\le 4$.
D. $x\le -2$ or $x\ge 4$.

5. The solutions of $x+\dfrac{x-5}{2}<-1$ and $3x+9>0$ are

A. $x>-3$.
B. $x>-2$.
C. $x<1$.
D. $-3<x<1$.

6. The solutions of $-4x-3>5-2x$ or $5x-5>0$ are

A. $x>-4$.
B. $x>1$.
C. $-4<x<1$.
D. $x<-4$ or $x>1$.

7. The solutions of $2x+\frac{11x+6}{5}>-3$ or $9-2x<3$ are

A. $x>-1$. B. $x>1$.

C. $x>3$. D. $-1<x<3$.

8. The solutions of $-4x\le 12\le 3x$ are

A. $x\ge -3$. B. $x\ge 0$.

C. $x\ge 4$. D. $-3\le x\le 4$.

9. If m is a positive integer satisfying the inequality $9-2m<3+m$, then the least value of m is

A. 0. B. 1.

C. 2. D. 3.

10. If n is a negative integer satisfying the inequality $12+n\ge 4-3n$, then the greatest value of n is

A. 0. B. –1.

C. –2. D. –3.

11. It is given that u and v are real numbers such that $uv<0$. Which of the following must be true?

I. $u-v>0$

II. $\frac{1}{u^2}+\frac{1}{v^2}>0$

III. $\frac{u}{v}<0$

A. I only

B. II only

C. I and III only

D. II and III only

12. It is given that $u > v$ and $k > 0$. Which of the following must be true?

I. $\frac{u}{k} > \frac{v}{k}$

II. $k - u > k - v$

III. $ku^2 + kv^2 > 0$

A. I only

B. II only

C. I and III only

D. II and III only

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Tutorial 12 More about Polynomials

Key Points

Division Algorithm

If a polynomial $f(x)$ is divided by a polynomial $p(x)$, the quotient $Q(x)$ and the remainder $R(x)$ obtained have the following relation:

dividend $\equiv$ divisor × quotient + remainder

$$f(x) \equiv p(x) \cdot Q(x) + R(x),$$

where the degree of $R(x)$ is less than that of $p(x)$.

For example, when a polynomial $f(x)$ is divided by $x + 1$, the quotient is $x^2 + x + 1$ and the remainder is 4, then $f(x) = (x + 1)(x^2 + x + 1) + 4 = x^3 + 2x^2 + 2x + 5$.

Remainder Theorem

(a) **(i)** When a polynomial $f(x)$ is divided by $x - a$, the remainder is equal to $f(a)$.

(ii) When a polynomial $f(x)$ is divided by $x + a$, the remainder is equal to $f(-a)$.

(b) **(i)** When a polynomial $f(x)$ is divided by $mx - n$, the remainder is equal to $f\left(\frac{n}{m}\right)$.

(ii) When a polynomial $f(x)$ is divided by $mx + n$, the remainder is equal to $f\left(-\frac{n}{m}\right)$.

For example, when $f(x) = x^3 + 2x^2 + 3x + 1$ is divided by $x - 1$, the remainder is $f(1) = 1^3 + 2(1)^2 + 3(1) + 1 = 7$.

Factor Theorem

Consider a polynomial $f(x)$.

(a) **(i)** If $f(a) = 0$, then $x - a$ is a factor of $f(x)$.

(ii) Conversely, if $x - a$ is a factor of $f(x)$, then $f(a) = 0$.

(b) **(i)** If $f\left(\frac{n}{m}\right) = 0$, then $mx - n$ is a factor of $f(x)$.

(ii) Conversely, if $mx - n$ is a factor of $f(x)$, then $f\left(\frac{n}{m}\right) = 0$.

For example, let $f(x) = x^3 - 6x^2 + 11x - 6$. Since $f(1) = 0$, $x - 1$ is a factor of $f(x)$.

Revision Exercise

Short Questions

1. Let $f(x) = 2x^3 - 5x^2 - 37x + 60$.

(a) Show that $x - 5$ is a factor of $f(x)$.

(b) Factorize $f(x)$.

2. Let $f(x) = 8x^3 + 14x^2 + x - 5$.

(a) Show that $4x + 5$ is a factor of $f(x)$.

(b) Factorize $f(x)$.

3. Let $f(x) = x^3 - 8x^2 + 29x + k$, where k is a constant. When $f(x)$ is divided by $x - 1$, the remainder is –30.

(a) Find the value of k.

(b) Is $x - 4$ a factor of $f(x)$? Explain your answer.

4. Let $f(x)=4x^3+kx^2-96x-63$, where k is a constant. When $f(x)$ is divided by $x-2$, the remainder is –275.

(a) Is $x-7$ a factor of $f(x)$? Explain your answer.

(b) Someone claims that all the roots of the equation $f(x)=0$ are real numbers. Do you agree? Explain your answer.

5. Let $f(x)=3x^3+x^2+kx+4$, where k is a constant. It is given that $f(x)=(x-1)(ax^2+bx+c)$, where a, b and c are constants.

(a) Find the values of a, b and c.

(b) Someone claims that all the roots of the equation $f(x)=0$ are rational numbers. Do you agree? Explain your answer.

6. Let $f(x)=6x^3+kx^2+25x+12$, where k is a constant. It is given that $f(x)=(2x-3)(ax^2+bx+c)$, where a, b and c are constants.

(a) Find the values of a, b and c.

(b) Factorize $f(x)$.

7. Let $f(x)=(x+1)^2(x+h)+k$, where h and k are constants. When $f(x)$ is divided by $x+1$, the remainder is 9. It is given that $f(x)$ is divisible by $x+4$.

(a) Find the values of h and k.

(b) Find the remainder when $f(x)$ is divided by $x+2$.

8. Let $f(x)=(x-2)^2(x+h)+k$, where h and k are constants. When $f(x)$ is divided by $x-2$, the remainder is 3. It is given that $f(x)$ is divisible by $x-1$.

(a) Find the values of h and k.

(b) Someone claims that there is only one real root in the equation $f(x)=0$. Do you agree? Explain your answer.

9. Let $f(x)$ be a polynomial. When $f(x)$ is divided by $x+2$, the quotient is x^2-4x-3. It is given that $f(-2)=18$.

(a) Find $f(4)$.

(b) Factorize $f(x)$.

10. Let $f(x)$ be a polynomial. When $f(x)$ is divided by $2x+1$, the quotient is $8x^2+6x-7$.

It is given that $f\left(-\frac{1}{2}\right)=4$.

(a) Find $f\left(\frac{1}{2}\right)$.

(b) Solve $f(x)=0$.

11. **(a)** Find the quotient when $4x^3-x^2-23x+6$ is divided by x^2-x-6.

(b) Let $f(x)=(4x^3-x^2-23x+6)-(ax+b)$, where *a* and *b* are constants. It is given that $f(x)$ is divisible by x^2-x-6.

(i) Find the values of *a* and *b*.

(ii) Solve the equation $f(x)=0$.

12. **(a)** Find the quotient when $2x^3+3x^2-32x+15$ is divided by x^2+3x-4.

(b) Let $f(x)=2x^3+3x^2+(p-32)x+15+q$, where p and q are constants. It is given that $f(x)$ is divisible by x^2+3x-4.

(i) Find the values of p and q.

(ii) Factorize $f(x)$.

13. Let $f(x)=(x-a)(x-b)(x+2)-4$, where a and b are integers and $0<a<b$. It is given that $f(1)=11$.

(a) **(i)** Prove that $(a-1)(b-1)=5$.

(ii) Find the values of a and b.

(b) Let $g(x)=x^2-18x+28$. Mandy claims that all the roots of the equation $f(x)=g(x)$ are integers. Do you agree? Explain your answer.

14. **(a)** Find the remainder when $x^{1001}+x+5$ is divided by $x+1$.

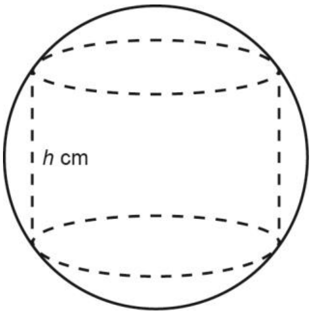
(b) **(i)** Using (a), or otherwise, find the remainder when $7^{1001}+12$ is divided by 8.

(ii) Joyce claims that $7^{1001}+9$ is divisible by 8. Do you agree? Explain your answer.

[Remarks: Q15 is relatively more challenging, students who target grade of Level 2 may skip it.]

15. **(a)** Let $f(x) = x^3 - 256x + 1224$. Find $f(-18)$.

(b) In the figure, a solid right circular cylinder of height h cm and volume V cm^3 is inscribed in a hollow sphere of radius 8 cm.



(i) Show that $V = 64\pi h - \frac{\pi}{4}h^3$.

(ii) If the volume of the cylinder is 306π cm^3, find the height(s) of the cylinder.
(Give your answer(s) correct to 3 significant figures.)

Multiple Choice Questions

1. Which of the following have $a-b$ as a factor?

I. $a^2-2ab+b^2$

II. a^2-b^2

III. $b(a-b)-a+b$

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

2. Let $f(x)=(3x+1)(x-1)+3x-1$. Find the remainder when $f(x)$ is divided by $3x-1$.

A. $-\frac{4}{3}$ B. $-\frac{3}{4}$

C. $\frac{3}{4}$ D. $\frac{4}{3}$

3. If k is a constant such that x^3+kx^2-x+30 is divisible by $x+2$, then $k=$

A. –30. B. –6.

C. 6. D. 30.

4. If $f(x)=x^3+3x-14$ is divisible by x^2+2x+k, then $k=$

A. –7. B. –2.

C. 2. D. 7.

5. Let $f(x)=x^{15}+4x+k$, where k is a constant. If $f(x)$ is divisible by $x-1$, find the remainder when $f(x)$ is divided by $x+1$.

A. –10 B. –5

C. 5 D. 10

6. Let k be a positive integer. When $x^{4k+1}-kx-k$ is divided by $x+1$, the remainder is

A. –1.

B. 1.

C. $4k-1$.

D. $4k+1$.

7. When $x^{2015}+x^{2014}+x^{2013}+...+x+1$ is divided by $x+1$, the remainder is

A. –1.

B. 0.

C. 1.

D. 2015.

8. Let k be a non-zero constant. When $x^2+5kx+6k$ is divided by $x-k$, the remainder is $-12k$. Find the value of k.

A. –3

B. –1

C. 1

D. 3

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Tutorial 13 Rate, Ratio and Variation

Key Points

Rate

A rate is a comparison of two quantities of different kinds by division. Rates have units.

For example, if Sam walks 6 km in 2 hours, then his walking speed = $\dfrac{6\text{ km}}{2\text{ h}} = 3\text{ km/h}$.

Ratio

(a) A ratio is a comparison of quantities of the same kind by division. Ratios have no units.

(b) For two quantities a and b, the ratio of a to b can be denoted as $a : b$ or $\dfrac{a}{b}$, where $a \neq 0$ and $b \neq 0$.

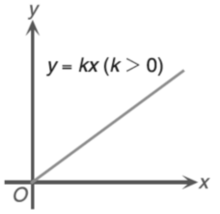
For example, if a drink is made by mixing 2 L of orange juice with 3 L of apple juice, then volume of orange juice : volume of apple juice = 2 : 3.

Variation

1. Direct variation

(a) If y varies directly as x, then $\dfrac{y}{x} = k$ or $y = kx$, where k is the variation constant and $k \neq 0$. In symbols, we write $y \propto x$.

(b) The graph of $y = kx$ (where $k \neq 0$) is a straight line passing through the origin with slope k.

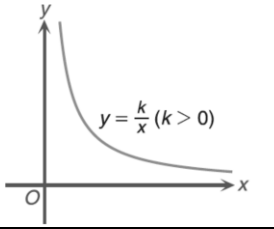


2. Inverse variation

(a) If y varies inversely as x, then $xy = k$ or $y = \dfrac{k}{x}$, where k is the variation constant and $k \neq 0$.

In symbols, we write $y \propto \dfrac{1}{x}$.

(b) The graph of $y = \dfrac{k}{x}$ (where $k \neq 0$) is a curve. It does not pass through the origin.



3. Joint variation

When a quantity varies directly as the product of two or more quantities, such relation among these quantities is called a joint variation. For example:

(a) If z varies jointly as x and y, then $z = kxy$, where $k \neq 0$. In symbols, we write $z \propto xy$.

(b) If z varies directly as x and inversely as y, then $z = \frac{kx}{y}$, where $k \neq 0$. In symbols, we write $z \propto \frac{x}{y}$.

4. Partial variation

When a quantity is the sum of two or more parts, where some of the parts may be constants, while other part(s) vary with other quantity/quantities, such relation is called a partial variation. For example:

(a) If z is partly constant and partly varies directly as x, then $z = k_1 + k_2 x$, where $k_1, k_2 \neq 0$.

(b) If z partly varies directly as x and partly varies inversely as y, then $z = k_1 x + \frac{k_2}{y}$, where $k_1, k_2 \neq 0$.

Revision Exercise

Short Questions

1. It is given that x varies directly as y and inversely as z. When $y = 8$ and $z = 1$, $x = 2$.

(a) Express x in terms of y and z.

(b) Find the value of x when $y = 8$ and $z = 3$.

2. It is known that y varies inversely as the square of x. When $x = 3$, $y = 20$.

(a) Express y in terms of x.

(b) If x is increased by 20%, find the percentage change in y. (Give your answer correct to 3 significant figures.)

3. It is given that $f(x)$ is the sum of two parts, one part is a constant and the other part varies as x^2. Suppose that $f(5) = 9$ and $f(10) = 84$.

(a) Find $f(x)$.

(b) Solve the equation $f(x) = 33$.

4. It is known that y is the sum of two parts, one part is a constant and the other part varies as x. When $x = 4$, $y = 134$; when $x = 5$, $y = 90$.

(a) Express y in terms of x.

(b) If x is a positive integer and $y > 120$, find all possible value(s) of x.

5. The parking fee of private cars in a car park is $\$P$ for x hours. It is given that P varies directly as x. When $x = 4$, $P = 72$.

(a) Express P in terms of x.

(b) Find the range of values of x when $P \leq 144$.

6. In an estate, the rent of a flat with floor area A m^2 is \$$R$. It is given that R is the sum of two parts, one part is a constant and the other part varies as A. When $A = 36$, $R = 14\,000$ and when $A = 45$, $R = 16\,700$.

(a) **(i)** Express R in terms of A.

(ii) Find the rent of a flat with floor area 62 m^2.

(b) If the rent of a flat is \$14 600, find the floor area of the flat.

7. The weight of a paper cardboard of perimeter x m is W g. It is given that W is the sum of two parts, one part varies as x^2 and the other part varies as x. When $x = 2$, $W = 440$; when $x = 3$, $W = 765$.

(a) Find the weight of a paper cardboard of perimeter 3.2 m.

(b) If the weight of a paper cardboard is 626.6 g, find the perimeter of the paper cardboard.

8. In a school, the time taken (T hours) to decorate a notice board varies directly as the area (A m^2) of the notice board and inversely as the square root of the number of student helpers (n) involved. When $A = 1.5$ and $n = 4$, $T = 3$.

(a) If 9 student helpers are involved in the decoration and the area of the notice board is 2.5 m^2, find the time required to decorate the board.

(b) It is given that the area of the notice board is 3.5 m^2. Is it possible that the time required to decorate the notice board with a certain number of student helpers is exactly 4 hours? Explain your answer.

9. In a factory, the cost of manufacturing a metal sphere of radius r cm is \$$C$. It is given that C is the sum of two parts, one part is a constant and the other part varies as r^2. When $r = 6$, $C = 138$ and when $r = 8$, $C = 152$.

(a) Find the cost of manufacturing a metal sphere of radius 14 cm.

(b) If the volume of a larger sphere is 27 times that of the sphere described in (a), find the cost of manufacturing the larger sphere.

10. Let the cost of painting a statue with surface area A m^2 be \$$C$. It is known that C is the sum of two parts, one part is a constant and the other part varies as A. When $A = 2$, $C = 327$ and when $A = 5$, $C = 555$.

(a) Find the cost of painting a statue with surface area 24 m^2.

(b) There is a smaller statue which is similar to the statue described in (a). If the volume of the smaller statue is $\dfrac{27}{64}$ times that of the statue described in (a), find the cost of painting the smaller statue.

11. **(a)** Let $f(x) = 4x^3 + 73x^2 + k$, where k is a constant. It is given that $x - 2$ is a factor of $f(x)$.

(i) Find the value of k.

(ii) Factorize $f(x)$.

(b) The cost of making a spherical sculpture of radius r m is \$$C$. It is given that C is the sum of two parts, one part varies as r^3 and the other part varies as r^2. When $r = 6$, $C = 6984$ and when $r = 9$, $C = 17\ 658$.

(i) Express C in term of r.

(ii) If the cost of making a spherical sculpture is \$648, find the radius of the spherical sculpture.

[Remarks: Q12 is relatively more challenging, students who target grade of Level 2 may skip it.]

12. It is given that $f(x)$ is the sum of two parts, one part varies as x and the other part varies as x^2. Suppose that $f(3) = -57$ and $f(8) = -112$.

(a) Find $f(4)$.

(b) $P(4, m)$ and $Q(18, n)$ are points lying on the graph of $y = f(x)$. Find the area of $\triangle PQR$, where R is a point lying on the x-axis.

[Remarks: Q13 is relatively more challenging, students who target grade of Level 2 may skip it.]

13. It is given that $g(x)$ is the sum of two parts, one part is a constant and the other part varies as x^2. Suppose that $g(2) = 18$ and $g(7) = 63$.

(a) Express $g(x)$ in terms of x.

(b) $P(0, m)$ and $Q(5, n)$ are points lying on the graph of $y = g(x)$. R is the foot of the perpendicular from Q to the x-axis. Find the area of $OPQR$, where O is the origin.

Multiple Choice Questions

1. If x and y are non-zero numbers such that $(3x + 5y) : (-x + 2y) = 4 : 1$, then $x : y =$

A. 3 : 7. B. 4 : 9.

C. 5 : 11. D. 8 : 15.

2. If x, y and z are non-zero numbers such that $2x = 3y$ and $y = 5z$, then $(x - y) : (y - z) =$

A. 3 : 4. B. 5 : 8.

C. 6 : 7. D. 8 : 11.

3. If a, b and c are non-zero constants such that $x(x-3a)+2b \equiv x^2 - cx + 3c$, then $a:b:c =$

A. $1:2:3$. B. $2:9:6$.

C. $3:2:1$. D. $9:2:3$.

4. The scale of a map is 1 : 50 000. If two schools are 5.4 cm apart on the map, then the actual distance between the two schools is

A. 0.027 km. B. 0.27 km.

C. 2.7 km. D. 27 km.

5. The actual area of a garden is 2500 m^2 and the area of the garden on a map is 100 cm^2. Find the scale of the map.

A. $1:25$ B. $1:50$

C. $1:500$ D. $1:2500$

6. The fee for booking a badminton court is \$110 for the first 2 hours and \$41 for each additional hour. Find the average hourly fee for booking a badminton court for 7 hours.

A. \$45 per hour B. \$48 per hour

C. \$52 per hour D. \$55 per hour

7. Winston runs for 50 minutes and his average running speed is 4.5 m/s. If his average running speed for the first 20 minutes is 3.6 m/s, find his average running speed for the last 30 minutes.

A. 2.1 m/s B. 5.1 m/s

C. 5.4 m/s D. 5.9 m/s

8. If 1 Chinese Yuan is equivalent to 1.19 Hong Kong dollars and 1 United States dollar is equivalent to 7.78 Hong Kong dollars, how many Chinese Yuan are equivalent to 50 United States dollar? Give your answer correct to the nearest Chinese Yuan.

A. 327 B. 368

C. 389 D. 463

9. It is given that z varies directly as x and inversely as the cube of y. When $x = 3$ and $y = 2$, $z = 24$. When $x = 5$ and $y = 4$, $z =$

A. 5. B. 20.

C. 64. D. 320.

10. It is given that z varies directly as $\sqrt{x}$ and inversely as y. Which of the following must be constant?

A. $yz\sqrt{x}$ B. $\frac{yz}{\sqrt{x}}$

C. $\frac{z\sqrt{x}}{y}$ D. $\frac{y\sqrt{x}}{z}$

11. It is given that z varies as x^2 and y. If x is increased by 10% and y is decreased by 20%, then z

A. is increased by 4.9%.

B. is increased by 8%.

C. is decreased by 3.2%.

D. is decreased by 12%.

12. It is given that y varies directly as x^2 and inversely as $\sqrt{z}$. If y is increased by 80% and z is decreased by 36%, then x

A. is increased by 20%.

B. is increased by 44%.

C. is decreased by 69%.

D. is decreased by 87%.

